Large deviations for Brownian motion

We would like to establish large deviation results for Brownian motion. The first difficulty is that the space of continuous trajectories is not compact so the theory developed during the course cannot be directly applied.

a) Look at the paper

http://www.math.tau.ac.il/~tsirel/Courses/LargeDev/lect9.pdf

in particular at sections 9a,9c,9d and extend the large deviation principle to non-compact spaces $\mathcal{K}$ introducing the property of exponential tightness.

b) Look at the paper

http://www-math.cudenver.edu/~puhalski/ld/course_1.pdf

and in particular at Theorems 2.5, 2.6 and 2.7 to help you prove the following statement. Cosider the space $\mathcal{X} = C([0,1];\mathbb{R})$ and let μ_n be the law of the process $(B_t/n)_{t\in[0,1]}$ on $\mathcal{X}$ where B_t is a standard Brownian motion of unit variance i.e. $\mathrm{Var}(B_t) = t$. Then μ_n describe a rescaled Brownian motion on [0, 1] with variance $1/n^2$. Prove that the family $\{\mu_n\}_{n\geqslant 1}$ is exponentially tight and that it satisfy a large deviation principle on $\mathcal{X}$ with rate function

$$I(f) = \int_0^1 (f'(s))^2 \mathrm{d}s$$

if $f' \in L^2([0,1])$ and $I(f) = +\infty$ otherwise.