List of possible questions

1. (Sec. 2.2, 3.2 in [1]) Definition of (local and non-) continuous-time martingales and relation between the two. In case:

    i. (Exercise 2 Problem sheet 10) Example of a local martingale that is not a martingale.

    ii. (Exercise 3 Problem sheet 7) Criteria under which a local martingale is a supermartingale.

    iii. (Exercise 3 Problem sheet 7) Criteria under which a local martingale is a martingale.

2. (Sec. 1.1 in [3], Sec. 1.2 in [1]) Construction of Gaussian processes via Daniell-Kolmogorov's theorem.

3. (Sec. 2 in [3], Sec. 3.2 in [2]) Lévy construction of Brownian motion. Hölder-continuity of Brownian motion.

4. (Sec. 2.3 in [1]) Construction of the Poisson point process.

5. (Sec. 2.4 in [1]) Doob's regularity theorem and càdlàg martingales.

6. (Sec. 2.5 in [1]) Convergence theorems of càdlàg (super-, sub-) martingales.

7. (Sec. 2.7, 2.8 in [1]) Stopping time, hitting time and first entrance time. Definitions, criteria for a hitting/first entrance time to be a stopping time.

8. (Sec. 2.9 in [1]) Doob's optional sampling theorem in continuous time.

9. (Sec. 3.1, 3.2 in [1]) Construction of stochastic integrals. (Simple processes, Itô isometry, extension)

10. (Sec. 3.3 in [1]) Itô formula with sketch of the proof.

11. (Sec. 3.3 in [1]) Lévy characterisation of Brownian motion.

12. (Sec. 3.5 in [1]) Statement of Girsanov's theorem.

    i. Sketch of the proof, or

    ii. (Sec. 3.4 in [1]) application to Black-Scholes, or

    iii. (Sec. 3.5.1. in [1], Sec. 4 in [4]) Novikov's criterion and application to weak solutions for SDEs.

13. (Introduction and Sec. 1, 2 in [4]) Definition of strong and weak solutions to SDEs; definition of pathwise uniqueness and uniqueness in law. Relations among such concepts. Tanaka's example.

14. (Sec. 1 in [4]) Existence and uniqueness for SDEs with globally Lipschitz coefficients. In case:

    i. (End of Sec. 1 in [4]) Criteria for local existence.

    ii. (Sec. 1.1 in [4]) Example of explosive solutions.

    iii. (Exercise 2 Problem sheet 12) Example of an SDE not satisfying growth condition but with global existence.

15. (Sec. 3 in [4]) Itô-Tanaka formula.

16. (Sec. 3.1 in [4]) SDEs with reflection.

17. (Sec. 4.4 in [1]) Conditioning of Brownian motion:

      i. Brownian bridge.

      ii. Bessel process.

Bibliography

[1] Anton Bovier. Introduction to stochastic analysis. Lecture Notes, WS2017/2018.
[2] Francesco De Vecchi. Stochastic processes: brownian motion. Lecture Notes 11, SS2019.
[3] Massimiliano Gubinelli. Foundations of stochastic analysis: Brownian motion. Lecture Notes 1, WS2019/2020.
[4] Massimiliano Gubinelli. Foundations of stochastic analysis: stochastic differential equations. Lecture Notes 2, WS2019/2020.