Lecture 1 – Tue April 21st 2020 – 14:15 via Zoom – M. Gubinelli

Schedule: Tuesday 14.15-15.45 and Wednesday 8.15-9.45 (SemR 1.007) Online until further notice

Prerequisites

Basic knowledge of continuous time stochastic processes and some stochastic calculus, e.g. see my course ``Foundations of Stochastic Analysis" from the WS19/20 (link). No previous knowledge of Quantum Mechanics will be assumed.

Literature

Strocchi, F. *An Introduction to the Mathematical Structure of Quantum Mechanics: A Short Course for Mathematicians*. 2 edition. New Jersey: World Scientific Publishing Company, 2008.

+ course material

Note

Seminar on "The mathematics of Feynman path integrals" (by F. de Vecchi) (url)

Introduction

Millennium problems

1. Introduction to quantum mechanics (Strocchi)

   a. History and motivation for quantum mechanics

   b. Axioms (C^* algebras, GNS representation, Hilbert space setting)

   c. Heisenberg group and its representation, Von Neumann theorem, Schrödinger representation

   d. Dynamics and the Hamiltonian (time $t \in \mathbb{R}$) H self-adjoint operator (matrix) Unitary transformation on an Hilbert space $U(t) = e^{iHt}$. $U(t)U(s) = U(t+s)$. $H \geqslant 0$.

   e. Examples: harmonic oscillator, particle in a potential

2. **Euclidean quantum mechanics** ($t \rightarrow -it = \tau$ imaginary time) $\Rightarrow$ Probability ('70-'80) Nelson/Symanzik/…

   a. Wick rotation ($t \rightarrow -it = \tau$ imaginary time) and Feynmann–Kac's formula, Wiener measure and connection with free particles.

   $$U(t) \rightarrow e^{-H\tau}$$

   b. Eucledian axioms (with *reflection positivity*) and reconstruction theorem

   c. *Nelson's positivity*, uniqueness of ground state and stochastic processes

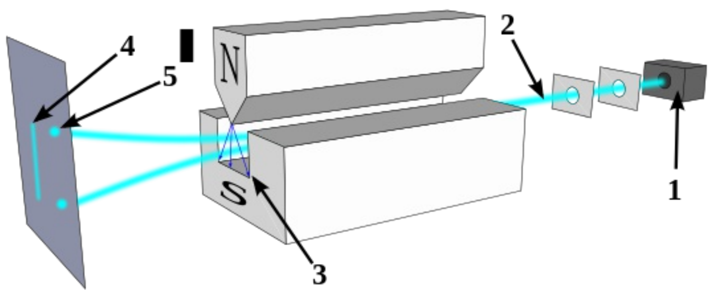
d. Particle in a potential and symmetric-stationary measure of SDEs with additive noise

e. Semiclassical limit ($\hbar \to 0$) and asymptotic expansion

f. *Introduction to Euclidean quantum* ***field*** *theory.* (special relativity)

$\Rightarrow \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \Rightarrow$ Quantum Y-M.

Stern–Gerlach experiment



1. Oven (source of Silver (47) atoms). 2. Collimated beam of atoms. 3. Non-homogeneous magnetic field. 5. Screen. 4. Classical result. 5. Real result.

One one of the 47 electron matters. Intrinsic magnetic moment $m \in \mathbb{R}^3$ $|m| = M$. It interacts with the magnetic field $B(x) \in \mathbb{R}^3$. Atoms are reflected differently according to the value of

$$\langle B, m \rangle_{\mathbb{R}^3} = B_z m_z$$

with $B = B_z \hat{z}$ and $m_z = \langle m, \hat{z} \rangle$.

Classically: one expects that every atom has a random magnetic moment $m \in \mathbb{R}^3$ so the quantity m_z is distributed like a continuous random variable.

Quantumly:

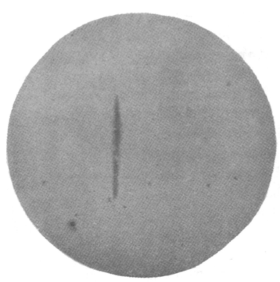


Fig. 2.

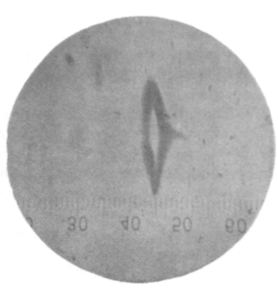
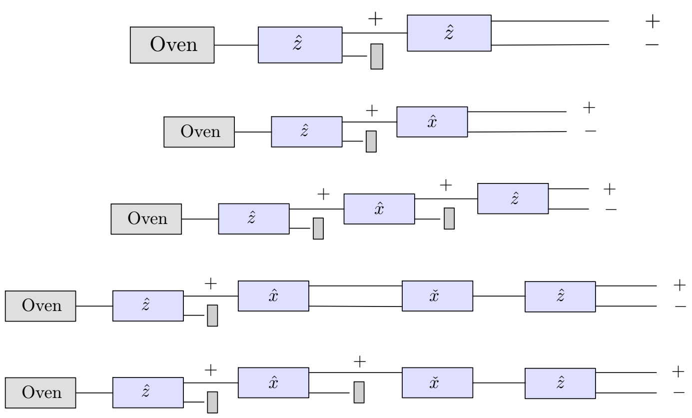


Fig. 3.

Conclusion: the electron intrinsic momentum (*spin*) is a quantum mechanical observable $m_z = \pm M$ (is *quantized*).



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Destructive measurement/interference. Non standard probability.

Wave behaviour (think about noise-cancelling headphones): sum of complex numbers.

Measurements do not commute: non-commutative algebra.
