RG (first try)

$$\int d\nu_P(\psi)\, e^{H(\psi)} = \int d\nu_{P_\gamma}(\psi)\, d\nu_g(\phi)\, e^{H(\psi+\phi)}$$

$$\overset{\text{int. out}}{=} \int d\nu_{P_\gamma}(\psi)\, e^{H_{eff}(\psi)} \quad \longleftarrow$$

$$\overset{\text{scaling}}{=} \int d\nu_P(\psi)\, e^{H'(\psi)} \quad \Longleftarrow$$

$$H'(\psi) = H_{eff}\left(\gamma^{-[\psi]}\psi(\cdot/\gamma)\right)$$

$$\frac{\int d\nu\, e^{H}\, \sigma}{\int d\nu\, e^{H}}$$

interaction

$$H(\psi) = \sum_A \int dx_A\, H(A,x_A)\, \Psi(A,x_A) + \underline{\underline{\text{const}}}$$

const disapp in finite volume

$$= \{H(A,x_A)\} \quad \text{seq int. kernels}$$



possible kernels

$|A| = \ell$ = # legs

$d(A) = p$ # derivatives $\Big\} \to \Psi(A,x_A) = \prod_{j=1}^{\ell} \partial^{n_j}_{\mu_j}\psi_{a_j}(x_j)$

$\ell$ even

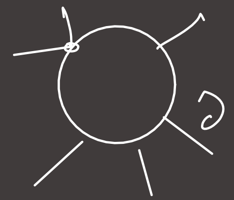
$n_j = 0, 1$

$x_j$ may coincide with $x_k$ $\quad \ell' \neq$ indep point.

ex $\quad \ell = 6$

$\ell' = 5$

$p = 1$




$$H_\ell = \{H(A,x_A)\}_{|A|=\ell}$$

$$H_{\ell,p} = \{H(A,x_A)\}_{\substack{|A|=\ell \\ d(A)=p}}$$

$\mathcal{B} = \{$ seq. of possible kernels $\}$

$H \in \mathcal{B}$ $\qquad H = \{H_{\ell p}\}_\ell$ even $\ell \geq 2$

$\qquad\qquad\qquad\qquad 0 \le p \le \ell$

RG: $\mathcal{B} \to \mathcal{B}$

$$H \to R(H) = D \circ I(H)$$

$I$ integrat

$D$ dilatation

• $I(H) = H_{eff}$

$$H_{eff}(B,x) = H(B,x) + \sum_{A \supset B} \int dx_{\tilde{B}}\, H(A, x_A)\, C(x_{\tilde{B}})$$

$g(x x')$



$$\sum_{B_1 \dots B_n} \sum_{A_1 \supset B_1 \dots A_n \supset B_n} \int H(\ )\, H(\ ) \quad C$$

• $D(H_{\ell p}(x)) = \gamma^{-D_\ell - p}\, \gamma^{d(\ell'-1)}\, H_{\ell p}(\gamma x)$

$$D_\ell = \ell[\Psi] - d = \ell\left(\frac{d}{4} - \frac{\varepsilon}{2}\right) - d$$

compare $H$ with $R(H)$ ~ norm

$$\| H(A) \|_w := \int_{x_1 = 0} dx_2 \dots dx_{\ell'}\, |H(A,x)|\, w(x)$$

$$w(x) = e^{c\left(\frac{\delta + x}{\gamma}\right)^{\sigma}}$$

Claim $\ell \geq 6 \Rightarrow D$ contraction [5]

proof

$$\| D\, H_{\ell,p}(A) \|_\omega = \gamma^{-D_\ell - p} \underline{\underline{\gamma^{d(\ell'-1)}}}\,.$$

$$\int_{x_1=1} \underline{dx_2} \dots dx_{\ell'}\ |H(A, \underbrace{\gamma x}_{y})|\ \omega(x)$$

$$= \gamma^{-D_\ell - p} \int_{y_1=0} dy_2 \dots dy_{\ell'}\ |H(A,y)|\ \underbrace{\omega\left(\tfrac{y}{\gamma}\right)}_{\leq \omega(y)}$$

$$\lesssim \gamma^{-D_\ell - p}\ \| H_{\ell p}(A) \|_\omega$$

[6]

$\ell \geq 6 \quad D_\ell > 0 \quad$ irrelevant

$\ell = 4 \quad D_4 = -2\varepsilon < 0 \quad$ rel

$\ell = 2 \quad D_2 = -\left(\frac{d}{2} + \varepsilon\right) < 0 \quad$ rel.

$\ell = 4,\ p = 1 \quad D_4 + 1 > 0 \quad$ irr

$\ell = 2 \quad p = 2 \quad D_2 + 2 = -\frac{3}{2} - \varepsilon + 2 > 0 \quad$ irr.

$d = 1, 2, 3$

$\ell = 2 \quad p = 1 \quad D_2 + 1 = -\frac{3}{2} - \varepsilon + 1 = -\frac{1}{2} - \varepsilon < 0$

rel.

---

① : starting point.

$$H_{st} = \nu \int dx\, (\psi \Omega \psi)(x) + \lambda \int dx\, (\psi \Omega \psi)^2(x)$$

$$\{ H_{2,0},\ H_{4,0},\ 0,\ 0 \ldots\ 0 \}$$

$\Rightarrow$ DoI get kernels with no decay

why $H_{eff}$ ?

ANS: bounds above for $\ell = 2, 4$
too naive

only local part relevant !



RG (correct version) add intermediate step

$$H \xrightarrow{I} H_{eff}$$

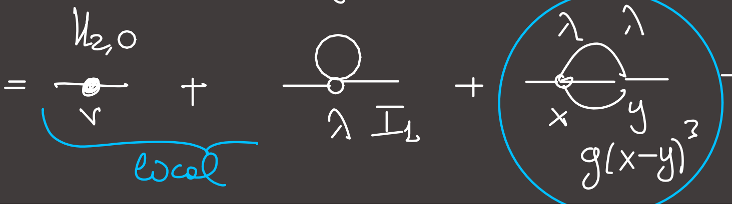
trimming
separate relevant terms

$$H_{eff} \xrightarrow{D} H'$$

first step $\quad H = \{ H_{2,0},\ H_{4,0},\ \ldots 0 \quad 0\ \ldots \}$

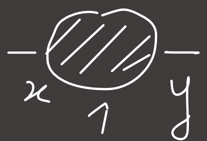

$I : H \to H_{eff}$. $\qquad I_1 = \int g(x)\, dx \qquad$ non local

$\ell = 2 \quad (H_{eff})_{2,0} =$




$+ \ldots$

general non local term

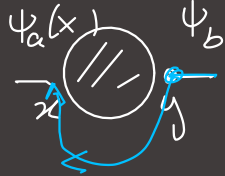


$$= \int K(x-y)\, \psi_a(x)\, \psi_b(y)\, \Omega_{ab}\, dx\, dy$$

1

one graph

turning (local): move one leg




interp id:

$$\psi_b(y) = \psi_b(x) + \int_0^1 \partial_t \psi_b(\overbrace{x + t(y-x)}^{x_t})\, dt$$

$$= \underline{\psi_b(x)} + \int_0^1 (y-x)_\nu\, \underline{\partial_\nu \psi_b(x_t)}\, dt$$



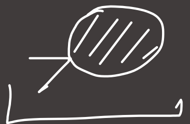
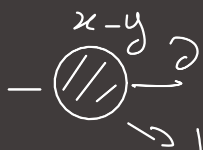
$$= \tilde{r} \int dx\, (\psi \Omega \psi)(x) \; +$$

$$+ \int dx\, dy \;\; K_\nu(x-y) \left( \psi(x)\, \Omega\, \partial_\nu \psi(y) \right)$$

$$\tilde{r} = \int dy\, K(y)$$

$K_\nu \simeq (x-y)_\nu K$

truly non local!

$=$  $+$ 

$$\| D - O^{\partial} \|_{\tilde{w}} = \gamma^{\frac{d}{2}+\varepsilon - 1} \| \; \|_w$$

rel.

$\downarrow D$

$$\| D > O \|_{\tilde{w}} \sim \gamma^{\frac{d}{2}+\varepsilon} \| > O \|_w$$

rel.

interp also the oth $\psi$


$$= \int dx\, dy\; K_\mu(x,y)\; \psi(x)\, \Omega\, \partial \psi(y)$$

symm

$$= \int dy\; (\psi \Omega \partial \psi)(y) \boxed{\int dx\; K_\mu(x,y)}$$

$(x-y)_\mu$

$$+ \int dx\, dy\; L_{\mu\nu}(x,y)\; \partial_\mu \psi(x)\, \Omega\, \partial_\nu \psi(y)$$

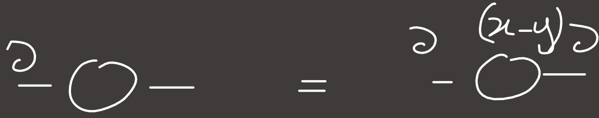

irrel.

$\Rightarrow \ell = 2,\ p = 0$


rel   irr.

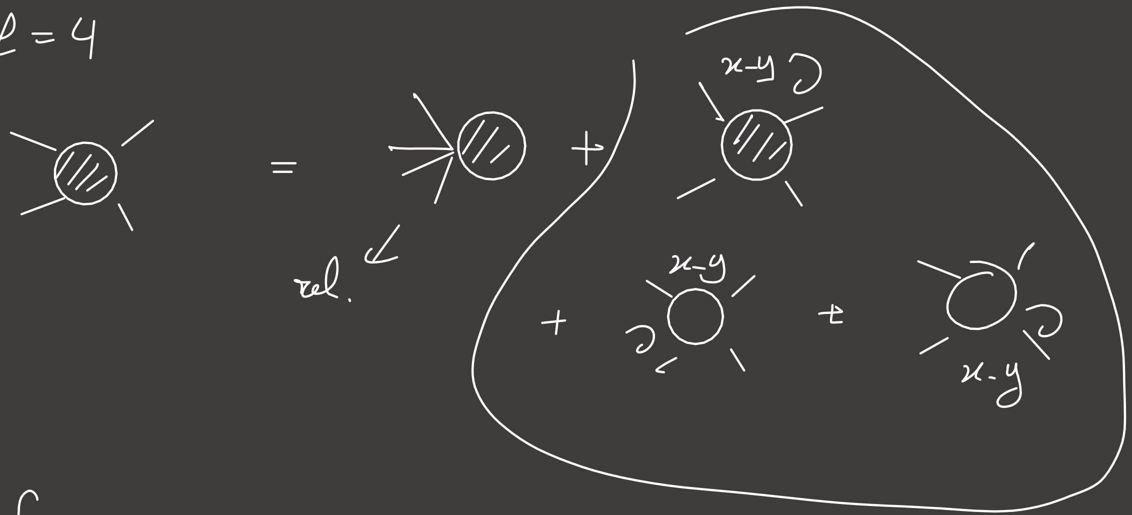


$\ell = 2 \quad p = 1$




irrel.

$\ell = 4$




irr

$$\int K_\mu\; \psi\psi\psi\, \partial_\mu \psi$$

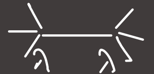
L13

$K_\mu \xrightarrow{I} K_\mu + \_ \; \_ \; \_$

$T($  $)$

$\| D($  $) \|_\omega \leq \underbrace{\gamma^{-D_4 - 1}}_{<1} \underbrace{\| \cdot \|_\omega}$

---

$\ell = 6 \quad (V_{eff})_{6,0} =$  $+$ oth terms

semi local

L14

trimmed seq = seq of kernels with add constr

$\underline{\ell = 2}$ $\quad H_{2,0}$ $\underline{\underline{\text{local}}}$ 

$H_{2,1} = 0$

set $H_{2L} := H_{2,0}$ loc

$H_{2R} := H_{2,2}$

---

$\underline{\ell = 4}$ $\quad H_{4,0}$ local 

set $H_{4L} := H_{4,0}$

$H_{4R} = \{ H_{4,p} \}_{p \geq 1}$

$l=6$ $\quad H_{6SL}$ semiloc of the form

$$\int \psi\psi\psi(x)\, \chi(x-y)\, \psi\psi\psi(y)$$

$H_{6R}$ all the others

---

$l \geq 8$ $\quad H_{l,p}$ as usual.

---

TL trimmed list $= \{2L, 2R, 4L, 4R, 6SL, 6R, 8, 10, \dots\}$

trimmed seq $\{H_{2L}, H_{2R}, H_{4L}, \dots\}$
$\{H_l\}_{l \in TL}$

$B_T$ = set of trimmed seq.



starting seq $H_{st} = \{H_{2L}, H_{4L}, 0\ 0\ 0\ 0\} \in B_T$

$$H_{eff} = I(H_{st}) \notin B_T$$

trimming $\quad T(H_{eff}) \in B_T$

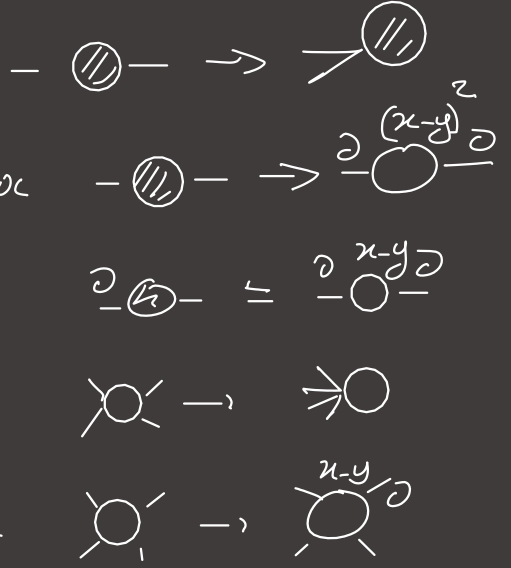
$T^{2,0}_{2L}(H_{2,0})$ p. local

$T^{2,0}_{2R}(H_{2,0})$ truly non loc

$T^{2,1}_{2R}(H_{2,1})$ "

$T^{4}_{4L}(H_{4,0})$ local

$T^{4}_{4R}(H_{4,0})$ truly non loc



L17

$$(H_{eff})_{6,0} = \underbrace{\overset{\mathcal{K}(x-y)}{\rightarrow\!\sim\!\sim\!\leftarrow} + \overset{H_{4L}\quad H_{4L}}{\underset{\lambda\quad\lambda}{\rightarrow\!-\!\leftarrow}}}_{H_{6SL}} + \underbrace{\text{oth terms}}$$

$$\bar{T}^{6}_{6SL}(H_{eff}) \qquad T^{6}_{6R}(H_{eff})$$

$$T(H)_\ell = H_\ell \qquad \forall\ \ell \geq 8$$

---

$$RG : B_T \to B_T$$

$$H \to R(H) = D \circ T \circ I(H)$$

$$H \in B_T \xrightarrow{I} H_{eff} \notin B_T \xrightarrow{T} T(H_{eff}) \in B_T$$

$$\xrightarrow{D} R(H) \in B_T$$

L18

$$\|D(H_{4L})\|_\omega = \gamma^{2\varepsilon} \underbrace{\|H_{4L}}_{\lambda}\|_\omega \qquad \text{rel.}$$

$$\|D(H_{4R})\|_\omega \leq \gamma^{2\varepsilon \underset{\substack{\uparrow \\ 1\text{ der}}}{-1}} \|H_{4R}\|_\omega \qquad \text{irr.}$$

$$\|D(H_{2L})\|_\omega = \gamma^{d/2+\varepsilon} \|\underset{\substack{\uparrow \\ \nu}}{H_{2L}}\|_\omega$$

$$\|D(H_{2R})\|_\omega \leq \gamma^{\frac{d}{2}+\varepsilon \underset{\substack{\uparrow \\ 2\text{ der.}}}{-2}} \|H_{2R}\|_\omega \qquad \text{irr.}$$

---

<u>warning</u> timing of gen errors!

$$\|D\, \overset{(x-y)^2}{\overline{T_{2R}}}(H_2)\|_\omega \simeq \gamma^{d/2+\varepsilon \underset{\substack{\uparrow \\ 2\text{ deriv.}}}{\cancel{-2}}} \gamma^{\cancel{2}} \|H_2\|_\omega$$

we do not lose a gain

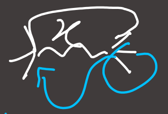
we gain only at the next RG step

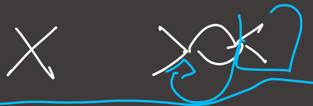
fixed point eq : $R(H) = D(T H_{eff}) = H$

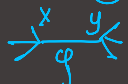
$\lambda \sim \varepsilon \quad v \sim \varepsilon$

2L $\quad v = \gamma^{\frac{d}{2}+\varepsilon}(v + \lambda I_1) + \sum_{\ell_1 \dots \ell_a} R^{\ell_1 \dots \ell_n}(H \dots H)$


4L $\quad \lambda = \gamma^{2\varepsilon}(\lambda + \lambda^2 I_2) + \dots + \dots$






6SL $\quad \mathcal{X}(x) = \gamma^{2d-6[\phi]}\left[\mathcal{X}(\gamma x) - 8\lambda^2 g(x\gamma)\right]$




$v_\ell = R(v_\ell)$

$\ell \neq 2L, 4L, 6SL$

solve $\mathcal{X}$ explicitly as a funct of $\lambda$

$\Rightarrow$ we can neglect $H_{6SL}$ from RG eq.

need only to consider the eq.

$\{H_{2L}, H_{2R}, H_{4L}, H_{4R}, H_{6R}, H_8 \dots, \}$

$$\|y\| = \max\left\{ \frac{|v|}{A_0 \delta}, \frac{\|H_{2R}\|}{A_0 \delta^2}, \frac{|\lambda|}{A_0^R \delta}, \frac{\|H_{4R}\|}{A \delta^2}, \frac{\|H_{6R}\|}{A \delta^3}, \sup_{\ell \geq 8} \frac{\|H_\ell\|}{A \delta^{\frac{\ell}{2}-1}} \right\}$$

$R : B_r \to B_r$ contraction