Stochastic Analysis SS2016. Sample exam questions

(please note that the list in not exhaustive!)

1. Difference between a weak and a strong solution to an SDE.
2. Various concepts of uniqueness. Consequences of path–wise uniqueness (Yamada–Watanabe)
3. What is a martingale problem for an SDE. Relation with weak solutions?
4. Under which sufficient conditions an SDE with path–dependent coefficients has a solution?
5. Give an example of an SDE with a weak solution but not a strong one (Tanaka).
6. How to represent continuous local martingales as time changed Brownian motion.
7. How to use time change to construct solutions to SDEs.
8. Example of non-uniqueness of weak solutions to SDEs. Relation with the Yamada–Watanabe theorem about uniqueness in one dimensions.
9. Properties of the complex Brownian motion under change of coordinates.
10. Structure of a absolutely continuous change of probability in a filtered probability space. State the general Girsanov transformation.
11. How to use Girsanov transformation to solve SDEs.
12. How to condition a diffusion not to leave a given domain or to reach a point at a given time.
13. What is the content of the martingale representation theorem. Some consequences?
14. Apply Boué-Dupuis formula to show Gaussian tails for Lipshitz functionals on the Wiener space.
15. State and give a sketch of proof of the Laplace principle for small noise diffusions.
16. What is the relation between the Laplace principle and the Large Deviation principle.
17. How to establish pathwise regularity of stochastic processes? example?
18. What is the relation between pathwise regularity and integrability of increments? example?
19. What is a stochastic flow for an SDE? Explain the main difference between the construction of a deterministic flow for an ODE and the construction of the stochastic flow of an SDE.
20. How to prove the pathwise injectivity of a stochastic flow.
21. How to prove the pathwise differentiability of a stochastic flow.
22. Describe the Stochastic Taylor expansion and the notion of weak and strong errors. Examples of weak and strong error estimates?
23. Describe numerical methods for SDEs and their strong and weak errors estimates.