Stochastic Analysis – Problem Sheet 6.

Tutorial classes: Mon June 6th in SemR 0.008. Philipp Boos <s6phboos@uni-bonn.de>.
Solutions will be collected at the beginning of the tutorial session. At most in groups of 3.

Exercise 1. Prove directly that the h-transform gives a transformation of martingale problems from $\mathrm{MP}(x_0, b, a)$ to $\mathrm{MP}(x_0, \tilde{b}, a)$ where $\tilde{b} = b + a\,\nabla(\log h)\nabla$. (That is, reproduce the argument of the notes without relying on the Itô decomposition of the process)

Exercise 2. Let $(X, \mathbb{P})$ be a solution of the Martingale Problem $\mathrm{MP}(x_0, b, a)$. Generalise appropriately the Girsanov transform to construct a measure $\mathbb{Q}$ under which the process X solves a martingale problem with a different drift. For simplicity, assume that all the necessary integrability conditions are satisfied. (Who takes the place of the Brownian motion?)

Exercise 3. Use Girsanov transform to prove that the weak solution of the SDE

$$\mathrm{d}X_t = b_t(X)\mathrm{d}t + \mathrm{d}B_t$$

where $b\colon \mathbb{R}_{\geqslant 0} \times C(\mathbb{R}_{\geqslant 0}; \mathbb{R}^d) \to \mathbb{R}^d$ is a bounded, previsible drift, is unique in law.