

*Sample exam questions*

Please note that the list is not exhaustive! The exam topics will be in general what has been discussed in the lectures, as recorded in the scripts, and the exercises proposed in the sheets. Below we highlight certain important points of the course which can be used to test one's understanding of the overall material.

1. Difference between a weak and a strong solution to an SDE, various concepts of uniqueness and relations among all these notions (Yamada–Watanabe's and Cerny's theorems). (Script 1, Script 2, Script 3, Script 4)
2. Give some examples of SDEs with the associated solutions (e.g. Sheet 1 Ex. 3, Sheet 2)
3. Levy's characterisation of multidimensional BM. (Script 4)
4. What is a martingale problem for an SDE. Relation with weak solutions? (Script 5)
5. What is the structure of one dimensional diffusions? (Script 8)
6. How to prove uniqueness of martingale problems? (Script 7)
7. Describe the continuous branching process. (Sheet 3 Ex 3)
8. Give the Feynman–Kac formula for diffusions. (Sheet 3 Ex 2)
9. How to represent continuous local martingales as time changed Brownian motion. (Script 6)
10. How to use time change to construct solutions to SDEs. (Script 5)
11. Example of non-uniqueness of weak solutions to SDEs. Relation with the Yamada–Watanabe theorem about uniqueness in one dimensions.
12. Give an example of an SDE with a weak solution but not a strong one (Tanaka).
13. Structure of an absolutely continuous change of probability in a filtered probability space. State the general Girsanov transformation. (Script 9)
14. How to use Girsanov transformation to solve SDEs. (Script 13)
15. How to prove uniqueness in law for diffusions with drift of linear growth. (Script 14, Sheet 8)
16. How to condition a diffusion not to leave a given domain or to reach a point at a given time (Script 12)
17. How to condition a diffusion to reach a point at a given time (Script 10).
18. What is the reflected Brownian motion? (Script 11).
19. What is the local time of a semimartingale, what properties it has? (Script 15, Script 16).
20. How to construct a reflected Brownian motion? What is the relation with the supremum of a Brownian motion? (Script 17).
21. What is the content of the martingale representation theorem. Some consequences?
22. Apply Boué–Dupuis formula to show Gaussian tails for Lipschitz functionals on the Wiener space. (Script 20).
23. What is the relation between the Laplace principle and the Large Deviation principle. State and give a sketch of proof of the Laplace principle for small noise diffusions. (Script 21, Script 22).
24. What is Schilder's theorem? Example of computation of a rate function? (Script 22).